Culturally Aware Mathematics Education Technology

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Abstract

Education and learning take place in a situation that is heavily influenced by the culture of the learners' context because this context affects a learner's cognitive processes in learning. Hence, to improve the conditions for learning, e-learning environments and their contents have to be presented and interact with the learner in a culturally appropriate way. Therefore, an e-learning system for cross-cultural usage has to adapt to the students' diverse cultural background. For the enculturation of the European platform for mathematics learning, ACTIVEMATH, a number of dimensions had to be adapted culturally: presentation of system and learning material, terminology, selection and sequencing of learning objects, interaction, and learning scenarios. The chapter describes ACTIVEMATH' enculturation: the empirical basis, computational model, and computational techniques for the cultural adaptation.

Topics: culturally-aware systems, groups' cultural specificities, culture and e-Learning, computational model and perspective

Solutions: adaptive systems, ontology engineering, semantic web techniques, knowledge representation techniques, user modelling techniques, system architecture

1 Motivation and Theoretical Background

One of the grand challenges in European TEL research focuses on technologies for learning that will be designed for culturally diverse and cross- and intercultural settings.

Enculturation is not only important for tutoring systems that teach cultural relationship, geography, culture, and behaving appropriately in other countries, etc. such as the cultural training system Alelo¹ but also for mathematics learning environments.

At first, mathematics education may seem to be an unlikely field for enculturation. However,

- mathematics education often follows the terminology (notions) of a particular book dominant in the region's education or of a field that applies mathematics, e.g., electrical engineering
- in the region's or field's mathematics education more often than not specific notations is used

 $^{^{1}}$ http://www.alelo.com/

- educationalists and teachers emphasize the cultural context and for teaching mathematics they follow the curriculum pre-scribed for their region and type of school
- educational psychologists and pedagogists emphasize the importance of posing authentic mathematical problems which are relevant in the student's life environment. This aspect is typical for a more constructivist learning paradigm [11, 2].

Theoretical Background. In addition, at the level of cognitive processes Piaget's theory of learning implies that learning heavily depends on the context and experience of the learner. Piaget's influencial theory of learning introduces two fundamental cognitive functional processes in learning: assimilation and accomodation [9, 10].

Assimilation and accommodation are two complementary processes of adaptation in the learner's mind through which awareness of the outside world is internalised. They are inseparable and exist in a dialectical relationship. In assimilation, what is perceived in the outside world (in the context) is incorporated into his/her mind without changing its structure. In accomodation, the mind accommodates itself to the evidence/context with which it is confronted and thus adapts to it.

An immediate implication of this theory is that the learner's context including its cultural determination is reflected in the learning process (and in its adaptation efforts). Assimilation requires more effort when the external and internal (mind) features differ more. Accomodation can include adaptation leading to more correct mathematical schemes/mind structures but also adaptation to cultural differences that are mathematically not as deep. Both types of adaptation yield cognitive load, but the cognitive load for a mathematically rather shallow adaptation is extraneous and may hinder learning.

From a theoretical point of view it has also be defined which (different levels of) cultural groups are relevant for enculturations in (mathematics) learning. It is not only the region/country/language that requires enculturation but also communities of practice (CoP), which can be a group inside another or a group orthogonal to a region/country/language group, e.g., chemistry or electrical engineering students. Wenger [17] acknowledges that (learning) practices are influenced by context(s) and that the internal dynamics of a CoP including its norms (part of a culture) are determined by the practices such as 'negotiation of meaning' (i.e., understanding of mathematical notions and notations), 'learning', as well as 'community actions' (e.g., book publishing, conferences), and 'differentiation from other communities of practice'. Any CoP produces abstractions, symbols, stories, terms, and concepts that reify something of its practice in a congealed form.

Note that even cultural variations whose nature appears to be linguistic such as notations and notions have a cultural (often historical) background. Reasons for such differences can originate from a dominant group of mathematicians in history and cultural ties to that group in certain countries, regions, and groups. A similar development (maybe not yet for mathematics) is observable today for communities of practice, e.g. users of email or SMS.

For cultural differences present in language-defined communities. One reason may be that in mathematics the history of schools, research, teacher education, popular books which influence the education culture go back to a language-defined culture that was present before todays countries existed.

Implications for Technology. A learning environment needs to respect the learner's (culturally influenced) mind structures to provide efficient learning support. More specific for the learning of mathematics is that more often than not the mathematical notations and even notions have to be adapted to the student's language, country, region and community of practice, i.e., to different levels of culturally specific groups. If a learning environment does

not adapt these notations and notions to what the student is used to, he/she has greater difficulties with assimilation and accomodation too and the cognitive load to get used to new notations and notions (or to new story contexts) reduces the cognitive potential available to the student for the actual learning. Therefore, examples and story exercises need to fit the real world experiences of the learner, e.g., American and British students would estimate/compute with miles and pound while continental Europeans would estimate/compute with gramm and meter.

Now, an e-learning system can be built only for one region or CoP, as it is often the case currently. However, in order to justify the huge effort for building and maintaining information technology for learning and for developing its content, e-learning systems need a large number of (potentially culturally diverse) users and, hence, potential cross-cultural usage. In Europe this need is particularly obvious. This is why the (European) platform for mathematics learning ACTIVEMATH has been enculturated a great deal.

Adaptation Dimensions. Technology-enhanced learning can be culturally adapted along several dimensions and these dimensions apply for the culturally-induced adaptation of learning environments for all kinds of domains, incuding mathematics:

- selection of learning objects (according to curriculum, familiarity of context, etc.)
- context of chosen examples and problems
- sequencing of learning objects (according to curriculum, learning scenario, etc.)
- presentation of material (according to language, design, etc.). For mathematics, the presentation of material includes approprite notions and notation
- preferred ways of interaction
 - choice of rote learning, application of algorithms, exploration, or critique
 - conversation style
 - tutorial strategies
 - learning scenarios.

Plan of the Paper. Our research has empirically collected, addressed, and realized the needed adaptation techniques and developed the underlying architecture and knowledge representations needed for enculturation. We consider cultural adaptations of

- mathematical notations
- mathematical notions
- input of formulalæ(interaction)
- choices of learning objects
- sequencing of learning objects and curriculum-dependent learning paths.
- last but not least, language-adapted user interface

This chapter addresses these dimensions for which the culture of the student, teacher, and author matters greatly. It elicits examples from European studies and describes the techniques used to adapt the platform for mathematics learning, ACTIVEMATH and its content, to the culture of the user. Some of the techniques can also be applied for mathematics systems with non-educational purposes.



Figure 1: Simplified architecture of ACTIVEMATH

2 Preliminaries: Basics of ACTIVEMATH

ACTIVEMATHIS a platform for mathematics learning. So far, it has mainly been used at high schools and universities in Germany and other European countries. Its functionalities include: presentation of content in one of the formats HTML, HTML+ MATHML, or PDF, depending on the user's choice; student modeling of the user's profile and competencies; assembling courses according to the user's learning goal, competencies, and language; interaction with the user, especially for exercises and examples (including student input diagnosis and feedback following selected tutorial strategies); (semantic) search for students, teachers, authors; authoring of semantic representations of mathematical content and of interactive exercises; group handling and information about student groups [8].

We briefly describe the relevant features, the knowledge representation and the relevant modules of ACTIVEMATH – student profile, presentation component, course generator. This sets the stage for explaining the techniques for culturally motivated adaptations.

2.1 Architecture

ACTIVEMATHhas a client-server, model-view-controller architecture with any browser as its most important client part, see Figure 1. The content is stored in internal or external data bases, and it is passed to components that need to access learning objects via a mediator that, in turn, accesses the various content bases as a web-service. These are shown at the left-hand side of Figure 1, which contains software that can register at a web-service. The components which realize adaptivities rely on data from the student model. After processing learning objects in any component the actual presentation of learning objects or interactions in the browser is prepared by a presentation engine.

2.2 Knowledge Representation and Presentation Component

ACTIVEMATH's knowledge representation of learning material is a semantic XML-representation for mathematical documents with educational extensions. This OMDoc itself is an extension of the semantic representation of OpenMath [1], an XML-representation of mathematical symbols (similar to MATHML-content). That is, the mathematical expression in any item/learning object are represented in OPENMATH.

The size of learning objects is fine-grained. Each learning object can have subitems in a number of languages. A learning objectis characterized (annotated) by metadata, some of which are culturally relevant. Metadata in OMDOC include: general administrative metadata (e.g., language, coverage); mathematical metadata defining types of mathematical items ('definition', 'theorem', etc.) and their relationships. These provide a basis for a domain ontology needed for course generation. In addition, pedagogical metadata serve the needs of course generation adapted to user's learning goals, personal preferences, and learning context.

Dublin Core metadata http://dublincore.org is used in ACTIVEMATH. It represents country/region-dependent priorities through the *coverage* metadata. For instance, a definition that is used within the region of Flanders is annotated with <Coverage country="be" region="Flanders"/>. *language* and *title* metadata are also used for enculturation in ACTIVEMATH.

Relationship between learning objects is encoded into metadata too. They are used for automatic sequencing of learning objects in courses by the course generator. Together with the concepts/fundamental learning objects they form an ontology and the 'prerequisite' and 'for' relationships can vary culturally.

2.3 Student Model

Currently, ACTIVEMATH' student model has a profiler and a competency model, see Figure 1. The profiler's values are initially set by the student and include 'language', 'country', and 'region' (e.g., Bundesland in Germany). See Figure 2.3 below in which the user's language is English (for the reader to understand), his country is Germany, and the region (of Germany) is Baden-Würtemberg. The values can be changed during his/her work with AC-TIVEMATHandthose changes lead to an immediate re-presentation of the student's material in language and in the cultural features specific for country and region. The 'field' value can be chosen from a number of fields (-of-study) such as electrical engineering, physics, chemistry, engineering, mathematics, etc., which represent field-specific CoPs.

2.4 Course Generation

The course generator, PAIGOS, assembles learning objects, bridging texts and dymanic items to a course adapted to the learner's needs [15]. During course generation, PAIGOS performs reasoning about learning objects. It uses its pedagogical knowledge and information represented in the domain ontology. For instance, it uses the information about prerequisites of definitions, and does general pedagogical reasoning about the prerequisites (such as which ones to present).

3 Cultural Variations Encountered and Solutions in AC-TIVEMATH

So far, we took an empirical perspective for the enculturation of ACTIVEMATH. That is, cultural adaptations focused on empirically encountered cultural adaptation needs which became obvious in studies for European projects. Not all but many of the adaptation dimensions mentioned above proved to be relevant in these studies, in particular adaptation of

| | Print Help Legal |
|--|--|
| ActiveMath | - |
| | Change User Data |
| Back Forward | |
| Account Data: | |
| Account name: | joe1 |
| Password: | Change password |
| | I accept the privacy policy. |
| Personal Data: | |
| How should ActiveMath call you? (e.g. first name or nickname) | Joe |
| What is your full name? | Joe Friendly |
| E-mail address: (optional) | joe@friendly.net |
| Language: | English 🛟 |
| Country: | Germany Region: Baden-Würtemberg 🛟 |
| This information will help Activ | veMath choosing appropriate content items: |
| What is your field? | Physics 🛟 |
| What is your educational level? | School, Secondary Education |
| How familiar are you with computers and the Internet? | somewhat familiar |
| Sa | ve Back |

Figure 2: User profile interface of ACTIVEMATH

- terminology notions and notations
- presentation of material
- selection of learning objects including choice of examples and problems
- sequencing of learning objects
- input of mathematical expressions.

In the empirical studies we found different levels of culturally determined communities (target groups) to which a learner, teacher, author can belong.

- language
- country
- region
- community of practice, possibly orthogonal to country and region.

All of these levels can influence the need of adaptation and, therefore, have to be represented in the student model, i.e., in the student profile.

3.1 Adapting Terminology

3.1.1 Empirical Basis.

We discuss only few examples from the long list of notions and notations including: permutations without repetitions, strictly increasing, instant slope, null sequence, Theorem of Thales, solution of quadratic equations, binomial coefficient, square root of -1, set of natural numbers, Least Common Multiple, logical notations.

Notions Some definitions and names of concepts differ in incompatible ways between language-based communities. For instance, the same concept is called *instant slope* in English and *momentane Steigung* in German but is described in geometric terms in French, as the *pente de la tangente* (slope of the tangent). Depending on the context, the English term "slope" is translated into Czech by different words:

- Přírůstek grafu funce (in the context of functions)
- Stoupání (in the context of hills, roads etc.)
- Směrnice přímky (in the mathematical context of lines a slope of a straight line)

Notion in the mathematics education differ in German regions, e.g., :

- (monoton) wachsend (in several states) vs . (monoton) steigend elsewhere
- Zentriwinkel (Thuringia) vs.Mittelpunktswinkel elsewhere
- Perihel (Thuringia) vs. Umfangswinkel elsewhere
- Basiswert for percentages (Nordrhein-Westphalen) vs. Grundwert elsewhere

Names Names of theorems can differ in different languages and cultures. for instance, the Inscribed Angle Theorem are called 'Theorem of Thales' in English and 'Satz des Thales' in German, while 'Theoreme de Thales' means an enlargement theorem in French.

As for region-related differences there are a number of examples for which schools in the German Bundesländer use different terminology, e.g., in Nordrhein-Westfalen the name 'Mitternachtsformel' is used whereeas in all other Bundesländer the same theorem is called 'Lösungsformel für quadratische Gleichungen'.

Notations The symbol for the binomial coefficient which is computed as $\frac{n!}{k!(n-k)!}$ is C_n^k in French speaking countries, $\binom{n}{k}$ in English speaking countries.

The symbol for the square root of -1 is *i* in most applications except in electrical engineering, where it is *j*

Units Units such as meter, inches etc. are special symbols which occur in applications of mathematics. The most frequent differences originate from the imperial system of measures vs. the metric system. An educational system needs to respect these differences too when presenting examples or problems.

In addition to the imperial/metric differences, units can also be used in a country-specific way. For instance, in Belgium, land is measured in *are* while this same unit is deprecated by the originators of the metric system, the Bureau des Poids et Mesures [12].

3.1.2 Models and Techniques.

The computational models underlying an adaptation of notions, names, and notations are mainly ontologies and metadata annotations which are encoded into the content's semantic knowledge representation. The techniques which use the knowledge representations to adapt belong to ACTIVEMATH' presentation pipeline.

Knowledge Representation. The symbol representation in Figure 3 shows metadata (in this case 'language') which characterize the different parts of the representation of a symbol's presentation/rendering including variables (in this case a and b). In the figure, the representation determines how to render the semantic symbol for binomial coefficient in presentation MATHML terms for English, Russian, and per default and what its precedence value is (for bracketing in a larger expression).

```
<symbolpresentation id="combinat1binomial_59_54" for="binomial">
 <notation precedence="1000" language="en">
   <math>
     <msubsup>
       <mo>C</mo>
       <mrow><mi am:precedence="0">b</mi></mrow>
       <mrow><mi am:precedence="0">a</mi></mrow>
      </msubsup>
   </notation>
 <notation precedence="1000" language="ru">
   <math>
     <msubsup>
       <mo>C</mo>
       <mrow><mi am:precedence="0">a</mi></mrow>
       <mrow><mi am:precedence="0">b</mi></mrow>
      </msubsup>
   </notation>
 <notation precedence="1000" language="default">
   <math>
         <mfenced>
     <mfrac linethickness="0">
       <mrow><mi am:precedence="0">a</mi></mrow>
       <mrow><mi am:precedence="0">b</mi></mrow>
        </mfrac>
        </mfenced>
   </notation>
 <OMOBJ>
   <DMA>
     <OMS cd="combinat1" name="binomial" />
     <OMV name="a" />
     <OMV name="b" />
   </OMA>
 </OMOBJ>
</symbolpresentation>
```

Figure 3: Rendering representation of the binomial symbol for several languages

Presentation The rendering process of mathematical formulæ works mostly using XSLT transformations. This transformation is performed in two stages [13] as illustrated in Figure 4.

- the first stage uses only the language/culture-relevant knowledge and applies XSLT to produce fragments. This is the place, where symbol presentations and notations are included.
- the second stage involves running the assembly of the content items into course pages and running other VELOCITY code in the presentation: the latter can employ all information relevant for cultural adaptation such as user profile and domain of the



Figure 4: ACTIVEMATH's rendering process

material. This is the place, where the actual rendering is finally adapted and where (culture-dependent) selection and sequencing are finalized.

Essentially, the XSLT transformation matches the OPENMATH term and replaces it by the associated culture-dependent rendering and replaces variables by their corresponding OPENMATH terms. All these features are collected and converted to an XSLT-template per symbol prototype. This template contains a condition on the language/culture to output the appropriate rendering for the language/culture chosen by the user. Then the template is converted to an XPATH expression. Then, the rendering process, which produces the browser material as delivered to the user, converts the OPENMATH encoded formulæ to one of the presentation formats, HTML, MATHML, or TEX using a series XSLT stylesheets [3].

For instance, for the notation for 'binomial coefficient' (with variables **a** and **b**) in Figure 3, a single representation can be rendered as variations depending on the user's language and culture. The different notations are built by the rendering process based on the information of the symbol representation in Figure 3.

The same rendering techniques can be used not only for modifying the presentation of symbols but also that of notions/names. Similar to symbols, the rendering facility fetches the title of the notion/name and places it during the preprocessing before the XSLT application.

3.2 Selection of Learning Objects

Empirical Basis. Students can understand examples and exercises from their life context (including culture) better than others. For instance, typically, students from a hilly region are more comfortable with an example or exercise addressing hill-walking tasks than students from a flat region. Similarly, examples and exercises with culturally appropriate units should be chosen (a choice between learning objects seems to be more appropriate than a conversion of units because the latter requires a reasonable adaptation of figures as well and it is complex to characterize what 'appropriate' means in those cases). Similarly, certain definitions are used for learning in a region or CoP but not in another. This is often based on the school curriculum of a region and applies, for instance, for definitions of continuity.

The example of *instant slope* in an English (and *momentane Steigung* in a German) curriculum with *pente de la tangente* in a French learning path. The French, geometry-

based definition needs to be trained by examples different from those in the English and German counterparts. Even the curricula of the different German states (Bundesländer) differ as shown in the table below. Hence, they can lead to different learning paths because all prerequisites of a concept may have been taught to students in one state but not for students in another state. For instance, in year 8 of a common type of school (Realschule) the content (sequence) to be taught in Baden-Würtenberg and Nordrhein-Westphalen is presented in the following table.

| Baden-Würtenberg | Nordrhein-Westphalen |
|-----------------------------|--------------------------|
| term rewriting | term rewriting |
| equations | data processing |
| perimeter and area | rectangles and polygons |
| linear functions | perimeter and area |
| systems of linear equations | percentage and interests |
| | linear functions |

Moreover, at places the concepts taught for a specific topic vary depending on the state. For instance, for Gymnasium (secondary school) 8th grade in Rheinland-Pfalz students learn about probability calculus (among others) how to determine a probability through simulation, whereas in 8th grade Gymnasium in Nordrhein-Westphalen they learn to use Pascal's Triangle.

Model and Techniques. The underlying models to cope with culturally determined selections are encoded into metadata such as Coverage (to characterize country and regional appropriateness) and Prerequisite relations. Moreover, the curricula of countries and regions can be encoded in groupings which can be used for course generation that includes selection and sequencing.

ACTIVEMATH's major intelligent component for realizing the culturally adaptive selection is the course generator, PAIGOS. It selects all learning objects as well as bridging texts whose conversational style may vary according to the user's culture

In order to explain how PAIGOS implements this cultural adaption, we need to describe how it selects and assembles learning objects to a personalized course. During course generation, PAIGOS reasons about learning objects. The underlying pedagogical knowledge is formulated with a vocabulary that is represented as an ontology of instructional objects (OIO) whose classes express different pedagogical purposes that learning objects can have [15]. For instance, during course generation PAIGOS might decide that at a particular place in the course (e.g., at instant slope) an exercise for a concept should be inserted. It also includes about domain-related metadata. All this translates to a query for a learning object, which basically consist of a metadata constraint. In order to realize the culturally implied differences in selection and sequencing, PAIGOS includes the location information of the student in the metadata constraint when collecting the prerequisites of a definition.

In the above example, the corresponding constraint (for a university student) is

(class fundamental) (relation isRequiredBy instantSlope) (hasCoverage France) (property hasLearningContext university),

which refers to the metadata for (IsRequiredBy), LearningContext and the culturally-induced locale (hasCoverage).

PAIGOS sends queries to the content storage(s), collects the identifiers of those learning object that fulfill the constraint, and continues to work with the resulting list.

PAIGOS dynamically assembles the content items to courses, taking information about the learner into account, e.g., his language, country/region, learning context as well as his competencies and preferences. The assembly, called course generation, is performed according to formalized pedagogical knowledge. This knowledge encodes e.g., which definition, examples and exercises to present, in which order, and how the course is structured in sections and subsection. In total, PAIGOS's pedagogical knowledge encompasses about 360 "rules".

PAIGOS's pedagogical knowledge is independent of the specific mathematical objects – however it uses information represented in the mathematical domain ontology. For instance, it uses the information represented in OMDOC about prerequisites of definitions, and does general pedagogical reasoning about the prerequisites (such as which ones to present). In ACTIVEMATH), each concept can have several definitions that differ in their Coverage metadata. If the Coverage metadata is not specified, the default ALL-countries is used. Upon a query, the content data base searches for the definition (more generally: learning object) whose Coverage corresponds to the user's country/region. If it exists, its identifier is returned. Otherwise, the identifier of a definition without Coverage metadata is returned. If there is none, an empty list is returned. These search results are used by PAIGOS which inserts the definition in the course that is appropriate from the Coverage perspective whenever possible.

3.3 Sequencing of Learning Objects

The educational practice of teaching mathematics exhibits large differences between cultures. The most obvious differences are caused by the curriculum standards which differ greatly from country to country or even from region to region. For instance, the notions of similarity and similar triangles occur in England at Key Stage 3 together with the notion of enlargement (and consequently with the intercepting lines theorem) whilst in France the intercepting lines theorem is taught without any reference to similarity of triangles which is taught about two years later in another mathematical context

Cultural differences due to curricula specific to countries or regions imply different sequencings of learning objects. Again, the course generator realizes the adaptation of learning object sequences.

Empirical Basis. In French curricula, the average slope – "pente de la sécante" (literally: slope of a secant) – is introduced via secants, i.e., geometrically, rather than analytically. So, a common French course does not use an equivalent of the English definition of the 'average slope' but other additional learning objects defining a secant. Similarly, the context in which notions such as *instant slope* (English) and *momentane Steigung* (German) are introduced, differs from the context for the French *pente de la tangente* (slope of the tangent) because the latter is based on geometric concepts which have other (learning) prerequisites than the prerequisites of instant slope.

Another bold difference in mathematical education can be observed for American/British and German/French learning paths: traditionally, American educational material provides plenty of examples before introducing formal definitions or theorems, while French or German educational paths prefer to first introduce the formal parts and only then the examples.

Model and Techniques. Automatic course sequencing with PAIGOS as well as dynamic sequencing of exercises with an EXERCISESEQUENCER [14] take country/regional differences into account.

| • | Arithmetic | | Relations | | Analysis | | Functions | | | Arithmetik | | Relationen | | | Analysis | F | Funktionen | • |
|-------|------------|-----|-----------|----------|----------|-----|----------------|------|------|------------|-----|------------|-----------|-----------------|------------|------------------|------------|-----|
| -0 | 0+0 | 0.0 | 00 | | | gcd | C ^D | | -0 | 0+0 | 0.0 | 00 | | <u>Σ</u> 0 | kgV((),()) | C ⁰ 0 | | |
| 0-0 | (0) | 0 | ₽ | <u>n</u> | 5 0-0 | Icm | | | 0-0 | 101 | | ₽Ū | <u></u> 0 | <u>р</u> 0-0 | 99T(D.D) | | | |
| | | | | | | | | | | | | | | | | | | |
| | | | | | | | | _ | | | | | | | | | | |
| Jcd (| 4,6) | | | | | | | | ggT(| 4,6 | | | | | | | | |
| | | | | | | | | | | | | | | | | | | |
| | | | | | | C | Ok Ca | ncel | | | | | | | (| Ok | Abbre | hen |

Figure 5: Formula editor palettes customized for different languages

The cultural profile of the user and the annotation of learning objects affect not only the selection of appropriate examples, exercises and texts but also the sequencing according to the prerequisite relationship. In the above example, a course generated for a French student presents the geometrical concept *tangente* first, followed by the notion of *pente de la tangente*. A course constructed according to the German way of sequencing the content shows the definition of (the graph of) a *function* first, and then the *momentane Steigung*. Thus, the courses generated for a French and a German student differs with respect to the presented definitions and their prerequisites. PAIGOS retrieves all learning objects that are related via the relation **prerequisite** and that fulfill the given constraint including the **coverage** of the learning objects. The coverage needs to correspond to the language, country, and region preferences of the user. In case definitions or theorems were found that fulfill the conditions, they are sorted with respect to the prerequisite relationship.

The described way of selecting prerequisites is one way of educational enculturation. Another means for taking into account cultural differences is to modify the pedagogical knowledge itself, e.g., to provide examples before introducing formal examples vs. first introducing the formal parts and only then the examples. The knowledge is represented declaratively and thus is adaptable. Currently, this knowledge encoding needs to be performed by an expert.

3.4 Enculturating Interaction

One of the interactions that has already been enculturated is the input of mathematical formulæ through an input editor. An input editor has to render symbols/expressions in the same way as they are rendered in the overall ACTIVEMATH. However, usually, a palette-based input editor is a rather independent software whose rendering is not necessarily the same as in the learning platform.

Empirical Basis. The variation of the look of mathematical symbols and expressions has been exemplified above already.

Model and Techniques. ACTIVEMATH has culture-adapted formula input methods. Students as well as authors can use a palette-based **input editor** for mathematical expressions in ACTIVEMATH. At the moment, ACTIVEMATH's formula editor supports multiple notations for different countries. This support is static, i.e., there are static palettes for each country (see Figure 3.4 palettes for Great Britain and Germany).

Our experiences show that **search** has to be enculturated too: not only the mathematical names and notions can differ between regions/countries/languages but also curriculum standards may differ in their vocabulary used to describe mathematical concepts or competencies as indicated in [4] and [6]. For instance, where the French curriculum mentions concrete theorems about proportions, the English curriculum mentions only the recognition of a transformation.

In the INTERGEO project, these differences gave rise to develop annotations to allow for cross-curriculum search. The annotations characterize topics, competencies, and educational levels which are part of a (hierarchical) ontology. For the search, the nodes of the ontology are used to expand basic text queries to queries for the corresponding ontology nodes and for nodes related to these. This makes relaxed search possible (through the hierarchy) and includes related resources in a language that is different from the one of the original query language and resource.

4 Related Work

A recent attempt to make the content of the Assistment system for mathematics aware of culturally determined units and names is described in [16]. It describes how conditional variables are used for certain symbols and names. This equals a simple approach in QTI (Question & Test Interoperability which is a standardized data format for online learning materials, mostly for quizzes and multiple choice exercises. see http://www.imsproject.org/question/). For instance, the variable 'sports' can have the instances 'baseball' (US), 'Cricket' (India), soccer' (EU). The technique for these modifications of the Assistment system's content uses an introduction of variables into exercises. For instance, additional variables are introduced for names, e.g., American idol names. The variables' instantiations then depend on rules for values, e.g., sport depends on country (if US then Baseball) or currency depends on country (if US then Dollar). As compared with our approach this simple variable instantiation is restricted to (symbol) names and used depending on the student's country only. It yields a syntactic replacement which does not preserve the semantics of symbols as it is important for ACTIVEMATH.

5 Conclusion and Future Work

We described some cultural adaptations that we empirically found necessary and described their technical solutions in ACTIVEMATH. We argued that these variations are culturally induced even if the group to which they apply is language-defined. The described variations are mostly for mathematical notions and notations and for presenting, selecting, and sequencing of learning objects.

5.1 Future Work

We expect the additional need for adapting/extending the choice of learning scenarios, tutorial strategies (including conversational style), design, and motivational features when we shall extend ACTIVEMATH' user community to non-European countries in Latin America, Russia, Ukraine, Moldavia, and China soon. For instance, the common way of Chinese education is based on rote learning. The Chinese Minstry of Education is aware that this needs a drastic change in order to support the transition of China to a country with more creative personnel and industry [18]. However, in order to introduce new types of exercises and learning scenarios in school is difficult because the rote learning is still the most effective way to pass university entry exams in China and is continued at the university. Given this current culture, it is not easy to prepare university students for alternative ways of learning. An additional problem fo this 'traditional' way of learning is the huge number of students which sets the stage for human-human teaching and thus determines much of this learning culture as well. After all, the number of university students grew to 400% during the last 6 years [5]. Overall, technology-enhanced learning makes sense to gradually change learning habits in China. And, a scenario for (Chinese) university entrance exams only with MCQs can make sense anyway.

So far we did not adapt to culture the diagnosis of student actions nor the updating mechanisms of the student model. Although we can imagine that this may be necessary at some point, still lack substantial experience. We are involved in experiments that will take place in Shanghai (China) to learn more about this issue.

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