Math-Bridge: Closing Gaps in European Remedial Mathematics with Technology-Enhanced Learning

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Abstract. Math-Bridge is an e-Learning platform for online courses in mathematics. It has been developed as a technology-based educational solution to the problems of bridging courses taught in European universities. Math-Bridge has a number of unique features. It provides access to the largest in the World collection of multilingual, semantically annotated learning objects for remedial mathematics. It models students' knowledge and applies several adaptation techniques to support more effective learning, including personalized course generation, intelligent problem solving support and adaptive link annotation. It facilities a direct access to learning objects by means of semantic and multilingual search. It provides rich functionality for teachers allowing them to manage students, groups and courses, trace students' progress with the reporting tool, create new learning objects and assemble new curricula. Overall, Math-Bridge offers a complete solution for organizing technology-enhanced learning of mathematics on individual-, course- and/or university level.

Keywords: bridging course, e-learning, mathematics education, adaptation, course generation, adaptive navigation support, e-learning platform, multilingual and cross-cultural aspects

1 Introduction

Many students enrolling into European colleges and universities lack mathematical competencies necessary for their studies, especially, in math-intensive engineering

[†] Dr. Erica Melis was the initiator and the coordinator of the Math-Bridge project. She passed away during the implementation of Math-Bridge.

and science disciplines (ACME, 2011). This is particularly problematic for a number of European countries (such as the Netherlands, Germany, Spain, UK, etc.) whose economies strongly depend on cohorts of well-educated engineers and who have been observing a steady decline in the number of engineering graduates in the last several decades (Becker, 2010). This leads to serious learning problems, and often causes students to drop out of their learning programs. For instance, drop-out rates for most engineering disciplines in Germany grew about 10% over the last 15 years and now stay at the level 25-35%. Drop-out rates for mathematics-intensive science study programs in German universities and colleges have also grown across the board to 15%-40% (Heublein, Schmelzer, & Sommer, 2006). Similar figures apply for several other European countries.

One source of this problem is that many students simply underestimate the necessary requirements to the mathematical background when they select a study program. This often results in confusion, frustration and lack of motivation to continue the study. On the other hand, the schools cannot always provide skillful teachers and enough content to prepare pupils for the university-level mathematics courses. Although the high drop-out rates cannot only be solved by focusing on mathematics education, an early opportunity to close the competency gaps and increase students' awareness about the requirements of university programs can help them to make an informed choice about their future studies, prepare for them properly and avoid potential detrimental effects on their motivation.

In order to facilitate practical efforts in this direction, and take a significant step towards improving European educational practices in the field of remedial mathematics, in 2009, a consortium of educators, mathematicians and computer scientists from nine universities and seven countries initiated the project Math-Bridge¹. This paper presents the details of this project. Section 2 underlines the problems of existing remedial courses offered by individual European universities and the directions of their improvement. Section 3 briefly summarizes the most important aspects of the approach implemented by Math-Bridge. Sections 3 to 6 provide the details of the Math-Bridge technology including the design and implementation of the developed elearning platform, characteristics of the accumulated collection of digital mathematical content and the main functionality available to the user of Math-Bridge, both students and teachers².

2 Problems of existing remedial courses

Educators across EU have long realized this set of challenges. The commonly accepted solution has become administering a dedicated bridging (or remedial) course that are usually offered for all students (or sometimes only to the students who fail the pre-

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² More details about Math-Bridge, as well as free access to the demo version of the platform is available at http://www.math-bridge.org

tests) in the beginning of their study program. Although these courses help to remedy the problem in the universities offering them, there are several drawbacks that cannot support implementation of a broader, cheaper and more effective solution.

2.1 Accessibility

To improve the situation, students in high school and first year higher education need interactive mathematics content for remedy that is not only available during remedial sessions at the university or on paper, but web-accessible. When or even before choosing a university/college they should be able to prepare for the requirements of their study subject.

For students who are not yet enrolled in a particular university program and also for students who want to migrate to another university in Europe, remedial content should be searchable and available not only locally, in the university, but on the web.

2.2 Support for cross-cultural and multi-lingual access to learning content

The target competencies that students are required to master often vary in description and names across countries, study programs and universities. It is not easy for prospective students, to get access to such requirements and learn them in advance. A solution is necessary that would provide a mapping between different institutions' curricula/requirements to a more generic set of competencies.

Mathematical notations often vary across countries, and in some cases, to the extent that it makes understanding of math content much more difficult for international students. Similar problems occur when the content is available only in one language. With increased student mobility and the development of the Bologna process, the number of international students in European universities grows. For international students, the transition to the university-level mathematics is aggravated by the fact that it has to be learnt in a language that they are not fluent in and that they did not use when taking the mathematics courses in school (Setati, Nkambule, & Goosen, 2011).

A related issue is the lack of appropriate metadata that would otherwise facilitate the discovery of learning content in a desired language.

2.3 Content reuse and interoperability

Even when the remedial content for mathematics is digitized it lacks the most basic forms of support for content discoverability, sharing, reuse, and interoperability. This means, the content cannot be easily found, altered and used by anybody but its authors. Content authoring for educational systems is a tedious and expensive procedure. In 2001, Downes estimated that development cost for a typical university-level course can be as high as \$100.000 (Downes, 2001). Therefore, it is crucial to ensure that the high-quality remedial content becomes easily available for the third-party users.

The problems often start with the formats in which this content exists. Many

teachers simply distribute their course materials as Word or TeX-documents, thus supporting no basic means for content reuse. Implementing it in HTML also provides little help in terms of improving its interoperability. Moreover, a large part of the content is compartmentalized by publishers or universities. It is not annotated with proper metadata, does not carry the semantic (mathematical) information that would be necessary to discover and retrieve the appropriate piece of content easily.

In order to realize the full potential of WWW as the infrastructure for content delivery and the platform where educational application are developed, the remedial content should be:

- implemented using standard-based semantic representation (XML and RDFbased formats for Web-content provide the basic means for content reuse across systems and contexts) (Buswell et al., 2004; Carlisle, Ion, & Miner, 2010; Kohlhase, 2006);
- 2. dissected into individual learning objects (this way, meaningful pieces of learning material can be discovered, and reused, the content can be reassembled according to the curricular of different courses, intelligent elearning applications can present to the students only the most appropriate learning resources);
- 3. provided with open-standard metadata, e.g. (ADL, 2001; IEEE, 2002; IMS, 2003) (which will facilitate content discovery, and enable adaptive access to the content based on its characteristics and the learning needs of the student);
- 4. authored in such a way that the meaning of mathematical symbols is decoupled from their rendering in the browser (this is essential for supporting multilingual and multi-cultural access to the content).

2.4 Interactivity

To support meaningful learning experiences and enhance students' engagement, remedial content collections must have interactive learning objects that can react to students' actions by providing proper feedback. Unfortunately, the dominant type of learning content available online is instructional texts and lecture slides.

2.5 Infrastructures for students, teachers and authors

Students' motivation, self-assessment, and performance can be improved by implementing collaborative learning scenarios in online educational environments, by maintaining timely assessments, by implementing an opportunity for self-organized assembly and annotation of content. This functionality is rarely available for them within the current bridging courses.

An adequate infrastructure for effective authoring of learning resources is also missing. Developing an individual learning object or assembling an entire course remains a very complex task and the current authoring tools provide little support to the authors.

A teacher of a bridging course must be able to easily manage all key aspects of the course, including student, course material, assessment tests, etc. A teacher should be

also provided with rich facilities for monitoring students' progress, detecting potential learning problems and intervening if necessary.

3 Math-Bridge: the Approach

The Math-Bridge project has addressed the problems outlined in Section 2 by applying a range of techniques from the fields of Intelligent Tutoring Systems, Adaptive Hypermedia, Semantic Web and Technology-Enhanced Learning. The result of these efforts is the e-Learning platform for bridging courses that has a range of unique features.

The developed Math-Bridge platform provides multi-lingual and multi-cultural semantic access (through semantic search, adaptive hypermedia and course generation) to remedial mathematics content, which adapts to the requirements of a learner and his/her subject of study. It brings together content from different European sources and offers it in a unified way. This access is provided through an online Pan-European learning service for remedial mathematics, which is built by collecting appropriate learning resources, extending them in terms of structure and multi-linguality and making them useful and easy-to-find. The extended formats of the content makes a wider use of standards and, hence, enables content reusability and transferability between different learning environments. In order to achieve this, an analysis of the target bridging competencies required for target subjects of study has been performed, the semantic and multi-lingual search software has been developed, the assessment tools have been implemented, and a range of remedy-scenario has been authored, which includes specific diagnostic means and decisions for the transition from school to higher education.

The solution implemented by Math-Bridge has been based on achieving several operational objectives:

- 1. To collect and harmonize high-quality remedial content developed by experts in bridging-level mathematics and make this content broadly accessible on the Web;
- 2. To enable cross-cultural and multi-lingual presentation of this content, thus, promoting its reuse across the borders;
- 3. To motivate the technological reuse of the content by implementing it in a shareable format and enriching it with metadata based on open standards;
- To offer different types of personalized access to the content, thus supporting multiple usage scenarios of the platform: from individual exploratory e-Learning to classroom-based knowledge training and testing;
- 5. To foster the adoption of the platform by increasing its usability not only for students, but also for other stakeholders, including teachers and university officials.

4 Math-Bridge: Content and Knowledge Base

4.1 Mathematical Content Collections

The Math-Bridge content base consists of several collections of learning material covering the topics of secondary and high school mathematics. They were originally developed for teaching real bridging courses by mathematics educators from six European Institutions: German Research Center for Artificial Intelligence, Open Universiteit Nederland, University of Saarland, Universities of Kassel/Paderborn, University of Vienna, Tampere University of Technology.

The content collections have been transformed through a sequence of operations in order to enable discoverability, interoperability and adaptability of learning objects constituting them. Fig. 1 presents the complete procedure of content transformation, step-by-step. The resulting database of remedial content is available as a collection of individual learning objects, transformed into the OMDoc format for mathematical documents (Kohlhase, 2006) and provided with metadata.



Fig. 1. Content transformation procedure

(lighter rectangles: evolution of the content; darker rectangles: stages of the procedure)

Compared to the majority of adaptive e-Learning applications, Math-Bridge supports a rich variety of learning object types. The OMDoc language used for representing content in Math-Bridge allows it to support a hierarchy of learning objects to describe the variety of mathematical knowledge.

On the top level, learning objects are divided into concept objects and satellite object. Satellite objects are the main learning activities, they structure the learning content, which students practice with: exercises, examples, and instructional texts. Concept objects have a dualistic nature: they can be physically presented to a student, and

s/he can browse them and read them; at the same time they are used as the elements of domain semantics, and, as such, employed for representing knowledge behind satellite objects and modeling students' expertise.

There are five main types of concept object available in Math-Bridge (see Fig. 2). Symbol is a special kind of concept objects. Symbols represent the most abstract entities in Math-Bridge, atomic mathematical concepts, which do not have content of their own. A symbol has a representation – a physical manifestation, which can be shown to a student and used in equations, but there is no actual learning content behind a symbol. Math-Bridge symbols are combined into an ontology, which is described in Section 4.3.

The rest of the concept objects model the most typical mathematical notions:

- definition is a statement, indicating meaning of one or several symbols;
- axiom is a postulate about one or several symbols;
- assertion is a statement about symbols; there can be several types of assertions, such as theorem, lemma, and corollary;
- proof represents a formal inference of an assertion and is always connected to the assertion it proves.



Fig. 2. Hierarchy of learning object types in Math-Bridge

The total number of learning objects in the Math-Bridge content base is almost 11000. Table 1 presents the details of this content base broken down by the types of learning objects available.

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Learning Object Type	Count
Interactive Exercise	5060
Instructional Text	2142
Learning Example	1563
Concept Definition	950
Assertion	634
Mathematical Proof	334
Axiom	36

4.2 Metadata Schema

The content collections of Math-Bridge consist of multiple learning objects of different types. Both, the collections and the individual learning objects are described in terms of their properties and attributes. The learning objects link to each other by multiple relations. Both, the attributes describing learning objects and the relations linking them together are specified via various metadata. The metadata elements can be divided into the following three categories:

- descriptive metadata used for administrative, cataloguing and licensing purposes; represented mainly using the Dublin Core standard (DCMI, 1999) (see Table 2).
- pedagogical metadata helping authors to specify multiple educational properties of learning objects; adopted and extended from IEEE LOM and IMS LD standards (IEEE, 2002; IMS, 2003) (see Table 3).
- semantic metadata connecting different learning objects to one another; partially relaying on OMDoc and SKOS standards (Kohlhase, 2006; Miles & Brickley, 2007) (see Table 4).

Overall, Math-Bridge metadata plays the core role in the overall architecture of the platform. It enables learning objects discovery, course composition, students' knowledge tracing and subsequent adaptation of the learning content.

Descriptive Meta	data					
title	name of the learning object					
date	date of creation (or last modification) of the learning object					
language	language of the content of the learning object					
creator	person, organization, or service, primarily responsible for creating the content of the learning object					
contributor	person, organization, or service, responsible for making contributions (e.g. translation) to the content of the learning object					
rights	rights held in and over the learning object, and provides the information regulating the learning object reuse and modification (uses Creative Commons schema)					

Table 2. Descriptive metadata elements of Math-Bridge

 Table 3. Pedagogical metadata elements of Math-Bridge

Pedagogical Meta	data					
field	subject of the course or major field of study of the audience of the					
	learning object					
coverage	geographical location the content of the learning object is suited					
	for					
difficulty	specifies how hard it is to work with (comprehend, complete) the					
	learning object for a typical target audience					
competency	category of mathematical skills (or cognitive activity) trained					
	(assessed) by the exercise					
competency	degree on which a corresponding competency is being trained					
level	(assessed) by the exercise					
learning time	approximate or typical time necessary to solve an exercise					
exercise purpose	educational purpose of an exercise (training vs. testing)					
exercise system	Web-service delivering the content of the exercise (if an exercise					
	is provided by an external Web-service)					
exercise type	the type of interactive elements used in the content of the exercise					
	in terms of the task that the learner has to perform in order to					
	complete them					

Table 4. Semantic metadata elements of Math-Bridge

Semantic Metada	ta					
broader/narrower	both directions of hierarchical relationships between symbols					
for	relationship between an exercise or an example and a concer					
	object that this exercise (or example) trains					
prerequisite	specifies, which concept items students have to master before					
	they can start working with a particular satellite object or start					
	learning a particular concept object					
introduction	specifies, that the satellite object introduces a concept object					
motivation	specifies, that the satellite object is used as a motivation for a					
	concept object					
elaboration	specifies, that a satellite object is an elaboration of a concept					
	object					
conclusion	specifies that a satellite object is a conclusion for a concept object					
lemma	specifies the relationships between two assertions, where one of					
	them acts as a proven supplementary proposition, necessary in					
	order to prove the other					
proof	specifies the relationships between a theorem/lemma and its proof					
corollary	specifies the relationships between an assertion and a lemma or					
	theorem, which it has been drawn from					
definition	specifies the relationship between a symbol and its definition					
reference	provides means for linking learning objects that mention each					
	other					

4.3 Math-Bridge Ontology

In order to model the domain of bridging mathematics, an ontology for the target subset of mathematical knowledge has been created. It serves as a reference point for all content collections and provides the source of the most abstract semantic metadata encoded through symbols. The ontology defines more than 600 concepts. It is used by the system logic for modeling students' knowledge, and adaptive course generation. The ontology is available in OMDoc and OWL³ (McGuinness & van Harmelen, 2004). Fig. 3 presents an extract of the Math-Bridge ontology opened in Protégé (Protégé, 2007).



Fig. 3. Math-Bridge ontology

4.4 Multilingual/Multi-cultural Aspect and Mathematical Notation Census

Math-Bridge content is available in seven languages: English, German, French, Spanish, Finnish, Dutch and Hungarian. The user can specify the language, in which s/he would like to read the content. To support multilingual students, individual learning objects can be translated on the fly. It is important to mention, that Math-Bridge translates not only the text but also the presentation of formulæ. Although mathematics is often called a "universal language", this is not fully true. In many countries, the same mathematical concepts use very different symbols (Libbrecht, 2010). In order to address this challenge, Math-Bridge separates the semantic and the presentation layer of math symbols. Inside the content, symbols are encoded using unambiguous entities, and when presented to the user, a correct notation is chosen

³ http://www.math-bridge.org/content/mathbridge.owl

based on the current language (Erica Melis, Goguadze, Libbrecht, & Ullrich, 2009). A public "notation census" has been conducted to document different notations of all symbols in all languages⁴. Table 5 presents the quantities of learning object translations available for each language.

Language	Number of Learning objects
English	9792
German	9792
Spanish	5099
Dutch	5484
French	4391
Finnish	5149
Hungarian	5905

Table 5. Different types of learning objects in Math-Bridge content collection

5 Math-Bridge: Technology-Enhanced Learning of Mathematics

The Math-Bridge platform provides students with multilingual, semantic and adaptive access to mathematical content. It has been developed based on the ActiveMath technology, and can be considered the next phase in the evolution of the ActiveMath intelligent tutoring system (Erica Melis et al., 2001; E. Melis et al., 2006; Erica Melis et al., 2009). Fig. 4 presents the dashboard of Math-Bridge. This is the entry point to the platform that every user sees after the login. Its interface consists of several widgets that provide access to different facilities available within Math-Bridge: regular and personalized courses, questionnaires, tests, and bookmarks.

If a user clicks on any of the courses, the student interface of Math-Bridge will launch (Fig. 5). It consists of three panels. The left panel is used for navigation through learning material using the topic-based structure of the course. The topics can have subtopics and can be folded and unfolded.

When one of the bottom-level topics is chosen, the content page associated with this topic is presented in the central panel. Each page can consist of multiple learning objects that a student can read. Exercise learning objects have a button "Start Exercise", which launches this particular interactive exercise in a separate tab. Students can also open new tabs when opening the results of the search and browsing through the learning object metadata.

Whenever a learning object is clicked on the content page, it gains focus. The right panel provides access to the details of the learning object currently in focus, as well as some additional features, such as semantic search and social feedback toolbox. It can also be used for on-demand individual translation of the learning object in focus to any language available for it, while the system interface and the rest of the content

⁴ http://wiki.math-bridge.org/display/ntns/Home

remain in the original language. This is one of the ways Math-Bridge supports international students. Alternatively, the entire systems interface and course material can be accessed in any of the supported languages too; the language can be specified at login.

MATH-BRIDGE		Dashboard	P S A Coput
Tips	~~	Dashboard	Group Algebra 101
Hello, John Math-Bridge Intro Math-Bridge Intro Mathematical States Mathematical States 0.00000000000000000000000000000000000	-	Courses Enriched VEMA Content (Kassel/Paderborn) Fraction calculation Introduction to Calculue tutContent collection	Differentiation using the Sum Rule course
		Questionnaires	Create a course
		Student Pre-Questionnaire Student Post-Questionnaire Tests	Overview - VEMA Content (Kassel/Paderborn) Accumulation points and isolated points - Introduction to Calculus Some linear equations - Introduction to Calculus
Legal Notes		Assessment Test	

Fig. 4. Math-Bridge dashboard

MATH-BRIDGE	Dashboard	P Image Ima
Table of contents <<	Differentiation according to the chain rule - Introduction to Calculus	» More
1. Basics	Learning Material Exercise X	Search 😵
2. Sequences, series, an	Application of the chain rule Compute the derivative of $f(x) = \cos(7 \cdot x^{10} - 10 \cdot x^7)$.	i Image: Social Application of the chain rule
3. Functions and relations	$\begin{aligned} f'(x) &= \left \cos(t \cdot x^{10} - 10 \cdot x^{1}) \cdot (t^{10} \cdot x^{9} - 70 \cdot x^{6}) \right \\ \text{No. Have another look at the chain rule. The first factor is g'(f(x)), not simply g(f(x)). So your factor \cos(t \cdot x^{10} - 10 \cdot x^{1}) is wrong.$	✓ Feedback ✓ Helpful
Difference quotients Introduction Into derivatives	$f'(x) = \frac{\sin(7 \cdot x^{10} - 10 \cdot x^2) - (70 \cdot x^2 - 70 \cdot x^2)}{\text{Not quite. Please check the signs. Also remember cos'(x)} = -\sin x .$	1 0 Comments
Differentiation rules Differentiation of po Differentiation accor Differentiation accor	$\int f'(x) = \frac{ x \sin(7 \cdot x^{10} - 10 \cdot x^2) \cdot (70 \cdot x^2 - 70 \cdot x^2)}{\text{Yes, that's it. You'll find a similar exercise with other values here.}$	▲ My Notes
 Differentiation accor Differentiation accor Differentiation accor 	(1) Your exercise is over, please close the tab.	Report problems Post a problem to our forums
Cvervlew over the d Legal Notes © 2012 Math-Bridge		

Fig. 5. Math-Bridge student interface

5.1 Tracing Students' Progress and Modeling Their Knowledge

Math-Bridge logs every student interaction with learning content. Actions, like loading a page, answering an exercise, accessing an individual learning object through the search tool are stored in the student's history database and help tracing and interpreting her/his learning progress.

The results of interactions with exercises (correct/ incorrect/ partially correct) are used by the student-modeling component of Math-Bridge to produce a meaningful estimation of the student's progress. The student model of Math-Bridge supports multi-layered representation of student's masteries. For every concept in the domain the model maintains a set of values estimating the probabilities that the student has mastered individual mathematical competencies associated with this concept. Every exercise in Math-Bridge is linked with one or several concepts (symbols, theorems, definitions etc.) and the competencies that the exercise is training for these concepts. A correct answer to the exercise is interpreted by the system as evidence that the student advances towards mastery and will result in the increase of probabilities for the corresponding concepts and competencies in this student's model. More details about the architecture and the implementation details of this student modeling mechanism can be found in (Faulhaber & Melis, 2008).

5.2 Personalized Courses

The course generator component of Math-Bridge allows students to automatically assemble a course optimized for their needs and adapted to their knowledge and competencies based on the current states of their student models. To generate a course, students need to select the target topics and a learning scenario. Several scenarios are available within Math-Bridge: a student can choose to explore a new topic, train a particular competency, prepare for an exam, master a previous topic or a assemble a course that will focus on the current gaps in student's knowledge. Each course type is generated based on a set of pedagogical rules defining the top-level structure of the course and the learning goals. The generation tool queries the student model and the metadata storage in order to assemble a didactically valid sequence of learning objects. Pedagogical metadata (such as exercise difficulty) and semantic metadata (such as prerequisite-outcome relations) play the central role in this process. More information about the Math-Bridge course generation tool can be found in (Ullrich, 2008).

On the interface level, generation of a single course is a simple 4-step process (see Fig. 6):

- 1. Click the button "Create a Course" in the "My Courses" widget of the Dashboard;
- 2. Choose one of the six course generation scenarios;
- 3. Select one or more target topics;
- 4. Name the course.

After that the generated course appears in the "My Courses" widget, and the student can access it the same way s/he accesses standard pre-defined courses.



Fig. 6. Course generation in Math-Bridge

5.3 Adaptive Navigation Support

The amount of content available within Math-Bridge is massive. Some of the predefined bridging courses consist of thousands of learning objects. In order to help students find the right page to read and/or the right exercise to attempt, Math-Bridge implements a popular adaptive navigation technique – adaptive annotation (Brusilovsky, Sosnovsky, & Yudelson, 2009). The annotation icons show the student how much progress s/he has achieved for the corresponding part of learning material. Math-Bridge computes annotations on several levels: each course in the Math-Bridge dashboard (Fig. 4, widget "Courses"), each topic within a course table of contents (Fig. 5, left panel) and each content page under a topic are provided with individual progress indicators aggregating student's learning activity on the corresponding level.

5.4 Interactive Exercises and Problem Solving Support

Interactive exercises play two important roles in Math-Bridge. First of all, they maintain constant assessment of students' knowledge thus providing the input for the student-modeling component. Second, they give students the opportunity to train mathematical competencies and apply in practice theoretical knowledge acquired by reading the rest of the content.

The exercise subsystem of Math-Bridge can serve multi-step exercises with various types of interactive elements and rich diagnostic capabilities. At each step, Math-Bridge exercises can provide students with instructional feedback ranging from mere flagging the (in)correctness of given answers to presenting adaptive hints and explanations (the central panel of the student interface shown by Fig. 5 presents an example of feedback produced by a Math-Bridge exercise).

Math-Bridge can automatically generate interactive exercises powered by external domain reasoner services. Currently, Math-Bridge uses a collection of IDEAS domain reasoners that provide stepwise diagnosis of students' actions and help generating advanced feedback and hints on every step of the solution (Heeren, Jeuring, van Leeuwen, & Gerdes, 2008).

The Math-Bridge platform also implements functionality for integrating third-party exercise services that maintain the full cycle of student-exercise interaction. As a result, students can access within Math-Bridge both, native Math-Bridge exercises and exercise served by remote systems. The integration is seamless for the student (Math-Bridge makes no difference in how native and external exercises are launched) and fully functional (Math-Bridge makes no difference in how students' interactions with native and external exercises are logged and interpreted by its modeling components). Currently Math-Bridge integrates two external exercise systems: STACK (Sangwin, 2008) and mathe online (Embacher, 2006).

5.5 Semantic Search of Learning Objects

In addition to navigating through the course topics, students have a more direct way to find learning objects of their interest – by using the Math-Bridge search tool. They can use default search based on simple string matching, advanced search that allows more precise specification of general search parameters (exact or practical matching, lexical or phonetic matching) and semantic search. The semantic search mode fully utilizes the advanced metadata schema of Math-Bridge. Students can specify the type of the desired learning object (e.g. only exercises), its target field of study (e.g. only easy exercises designed for physics students), etc. Fig. 7 presents the interface of the Math-Bridge search tool. The left part shows the results of simple search with the same word and the target type of learning objects restricted to assertions.



Fig. 7. Math-Bridge search tool

6 Technology-Enhanced Teaching of Mathematics

Math-Bridge offers teachers and university IT specialists a complete arsenal of tools necessary to setup, administer and teach online courses.

6.1 Content and Course management

Teachers can create their courses from scratch or reuse one of the existing tables of contents. They can design assessment tests, exams and questionnaires, and author individual learning objects and collections of new material.

6.2 User and Group Management

There are three categories of users in the system. Students can access learning content individually or as a part of a course. Teachers can manage their courses, including content visibility and student roster. Administrators have access to all aspects of Math-Bridge user management. They can change user parameters and rights, modify group membership, and assign a teacher to a course. Naturally, administrators can also do everything that other users can.

6.3 Course Monitoring with the Reporting Tool

It is easy for teachers to monitor students' progress within Math-Bridge: a dedicated reporting tool allows them to trace individual student's performance or results of the entire class. The reporting tool can also help in discovering potentially problematic

learning objects (e.g. an exercise that nobody has solved correctly). Fig. 8 presents an outcome example of an aggregated report on exercises. Overall, Math-Bridge provides teachers with about 10 different reports.

MATH-BRI	DGE				Dashboard			P Image Ima
Manage	~	Reports					×	>> More
Logged in as	: winter	Results are filtered by: "time This report is based on eve	erange" "excludeTutors" nts from users of all groups,					Search
Reports		Report for 19 exercises, bar	sed on 33 runs and 65 event	s.				🕼 About
laam		Exercise	Tries		Users	Times & Avera	los 🖉	⊿ Details
-	_	Exercise 1a mbase://BasWis/opgaven-	numAccesses:	1	numUsersAccessed: 1	totalTime (secs):	2	Type Aggregate Report - Exercises +
Groups		bij-les-1/exc_opgave1a	numAttempts:	1	numUsersAttempted: 1	avgTime	2	Course <al></al>
Server			numAttemptsGivenup:	0	numUsersSolved: 0	(secs):	0	Group <ai></ai>
			numAttemptsFinished:	1		(%):	Ŭ	Include tutors authors and admine
			numAttemptsSuccessful:	0				Time Range 2012-02-21 - 2012-03-20
		Exercise 4 mbase.//BasWis/powers-	numAccesses:	1	numUsersAccessed: 1	totalTime	18	View Report Export Data
		neg/exercise-powers-neg10	numAttempts:	1	numUsersAttempted: 1	avgTime	18	
			numAttemptsDropped:	0	numUsersFinished: 1	(secs):		
			numAttemptsGivenup:	0	numUsersSolved: 0	avgSuccess (%):	0	
			numAttemptsSuccessful:	0		(
		Exercise 1	numAccesses:	1	numUsersAccessed: 1	totalTime	4	
		mbase://BasWis/powers- neg/exercise-powers-neg2	numAttempts:	1	numUsersAttempted: 1	(secs):	·	
			numAttemptsDropped:	0	numUsersFinished: 1	avgTime (secs):	4	
			numAttemptsGivenup:	0	numUsersSolved: 0	avgSuccess	0	
			numAttemptsFinished:	1		(%):		
			numAttemptsSuccessful:	0				
		Exercise 2 mbase //Bas/Wis/nowers	numAccesses:	1	numUsersAccessed: 1	totalTime	6	
Legal Note	85	neg/exercise-powers-neg3	numAttempts:	1	numUsersAttempted: 1	(secs):		
© 2011 Math-	Bridge		numAttemptsDropped:	0	numUsersFinished: 1	avg I me (secs):	0	

Fig. 8. Math-Bridge teacher interface: reporting tool

7 Conclusion

Math-Bridge is a full-fledged e-Learning platform developed to help individual learners, classes of students, as well as entire schools and universities to achieve their real-life educational goals. Math-Bridge implements a number of advanced technologies to support adaptive and semantic access to learning content. Fostering the adoption of these technologies by the general public is the primary goal of Math-Bridge.

The platform has been evaluated with more than 3000 students from nine universities and seven European countries (Mercat, 2012). Math-bridge has been used under different scenarios: as the main learning platform in a distant course, as an online component in a blended course, and as a supplementary tool in a traditional course. Under such diversity of usage scenarios and educational settings, it has been confirmed that students learn with Math-Bridge and that they in general feel very positive about using the tool. Further experiments are required to detect other effects of using Math-Bridge for learning and teaching remedial mathematics.

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